Departure Dynamics of Ultracold Bose-Einstein Condensates on a 1D Lattice with Inhomogeneous Chemical Potential

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Bose Hubbard Hamiltonian

$$\mathbf{H} = -\sum_{i=0}^{L-1} \mu_i \mathbf{n_i} + \frac{U}{2}\sum_{i=0}^{L-1} \mathbf{n_i}(\mathbf{n_i}-1) - \omega \sum_{i,j=0}^{L-1} \left(\mathbf{b_i}^\dagger \mathbf{b_j} + \mathbf{b_j}^\dagger \mathbf{b_i}\right)$$

$\mathbf{b_j}, \mathbf{b_j}^\dagger$ bosonic annihil. and creation operators
$\mathbf{n_j}$ counting operator of site j

Working Hypothesis

- next neighbor hopping only
- $U \to \infty$
- μ_i depends on i

Conserved Quantities

- Total Energy ($\frac{\partial}{\partial t}\mathbf{H} = 0$)
- Number of Particles resp. Total Spin ($[\mathbf{H}, \mathbf{n_i}] = 0$)

Motivation

- Applications
  - Atoms in optical lattices
  - Application to superconductor theory expected (Mott-Transition [1])
  - Entanglement of ultra-cold atoms (Quantum Computing)
  - Linking small bulk material and substrate
- Fine-Tunable Realization:
  - Shown analogy to atoms in periodic laser fields (Dipole-Interaction) [2]
- Goals:
  - General understanding (so far few studies on this field)
  - Development of a protocol for "self-compression" of BECs

Outlook

- Pair annihilation & creation
- Different potential structures (e.g. periodic)
- Semi-open systems
- ...

Workframe and References

Work done within the framework of a "Stage M2" (Masters Thesis) supervised by Dragi Karevski at "Institut Jean Larmour" in the "Laboratoire de Physique des Matériaux" at Vandœuvre-les-Nancy.

References

[1] M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher. Boson localization and the superfluid-insulator transition. *Phys. Rev. B*, 40(1):546–570, Jul 1989.

[2] D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller. Cold bosonic atoms in optical lattices. *Physical Review Letters*, 81:3108, 1998.

[3] D. Karevski. Ising quantum chains, Nov. 2006.

[4] S. Sachdev. *Quantum Phase Transitions*. Cambridge University Press, 1999.

Theoretical Development

Interacting Bosonic System

$$|n_i\rangle = |1\rangle, |0\rangle \text{ with } \left[\mathbf{b_i}, \mathbf{b_j}^\dagger\right] = \delta_{i,j}$$

can be transformed to Interacting Spin Chain (XX-Model with inhomogeneous potential)

$$|\sigma_i^z\rangle = |\uparrow\rangle, |\downarrow\rangle \text{ with } \left[\sigma_\mathbf{i}^\mathbf{j}, \sigma_\mathbf{i}^\mathbf{k}\right] = 2\mathrm{i}\epsilon_{jkl}\sigma_\mathbf{i}^\mathbf{l} \text{ with } j,k,l = xyz$$

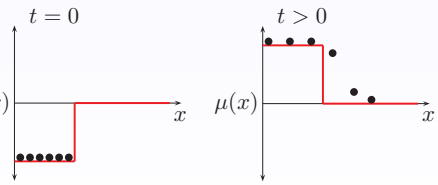
Jordan-Wigner-Transformation to Non-Interacting Fermions

$$\left|\mathbf{c_i}^+\mathbf{c_i}\right\rangle = |1\rangle, |0\rangle \text{ with } \left\{\mathbf{c_i}^+, \mathbf{c_i}\right\} = \delta_{i,j}$$

Quadratic Hamiltonian which can be diagonalized (cf. [4, 3] for a more detailed discussion of the formalism)

Protocol

At $t = 0$: Creation of a thermal initial state (shown $T = 0$) $t > 0$ Creation of the out-of-equilibrium situation by abruptly changing the underlying potential. Other forms than the step shown here can also be used.



Thermal States

Expectation values for many-body states:

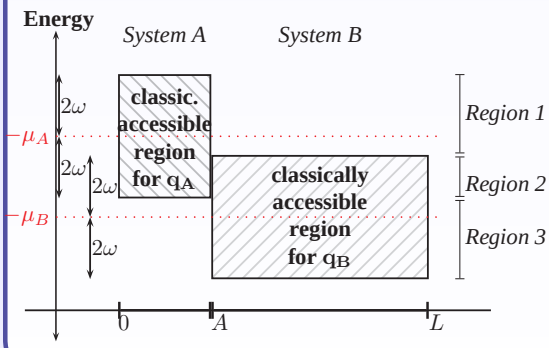
$$\langle \mathbf{Op} \rangle = \mathrm{Tr}\left\{\mathbf{Op}\,\frac{\mathrm{e}^{\frac{\mathbf{H}}{T}}}{\mathbf{Z}}\right\}$$

Many-Particle Dynamics

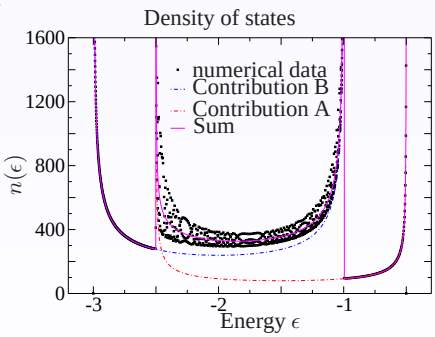
- Only time-development of two-operator correlators necessary (Wick's theorem)
- Expectation values of correlators can be written as matrix product

Step Potential - Spectrum

1-part. dispersion: $\epsilon_q = -\mu_i - 2\omega\cos(q_i)$, $\Re\{q_i\} = 0 \ldots \pi$



Accessible energy spectrum.

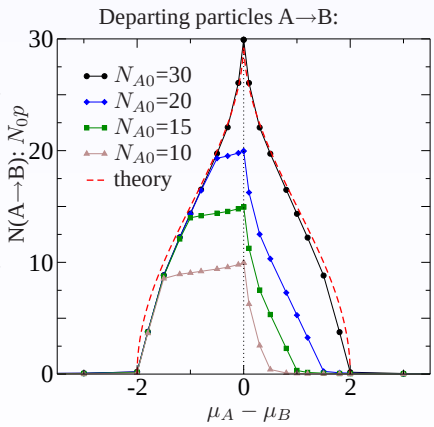


Parameters $\omega = 0.5$, $\mu_A = 1.5$, $\mu_B = 2$
$L = 1000$ $A = 250$

Step Potential - Self-Trapping

- The departure of particles is governed by the number of modes connecting the systems
- Corrections to an infinite system scale with $\frac{A^2}{L}$
- The higher the potential difference the less particles can leave the initial system
- Contrary to a classical system, the energy structure blocks particles even for a configuration of the potential classically favoring the departure of particles

Ongoing research in generalization of these techniques to other driving potentials.



$A = 30, L = 50, \omega = 0.5$