Meso-Scale Symmetries explain Dynamical Equivalence in Food Webs

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Introduction

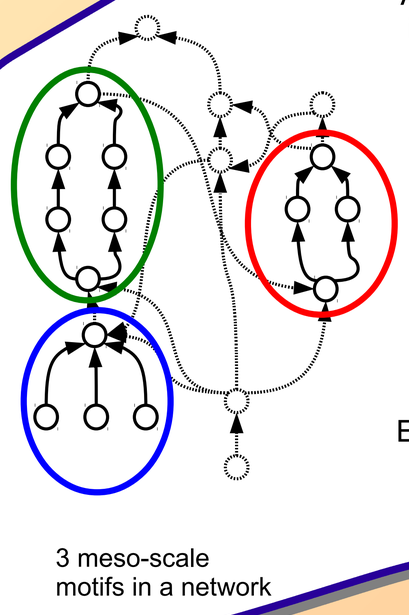
A goal of complex system research is to find implications of the interaction structure on the dynamics of a system.

►Local phenomena from single nodes are simple.

►Global phenomena can be solved with global methods (eigenvalue computation,...).

►How about meso-scale phenomena that involve a few nodes?

Example: Network motifs in ecological food webs.



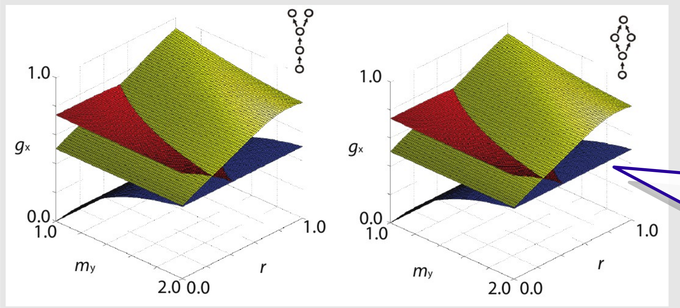
3 meso-scale motifs in a network

Motivation

►Describe dynamics too complicated for a node-by-node description, but where system size prohibits global methods.

►Understand why different food webs have the same bifurcation diagrams.

Identical bifurcation diagrams for two different topologies



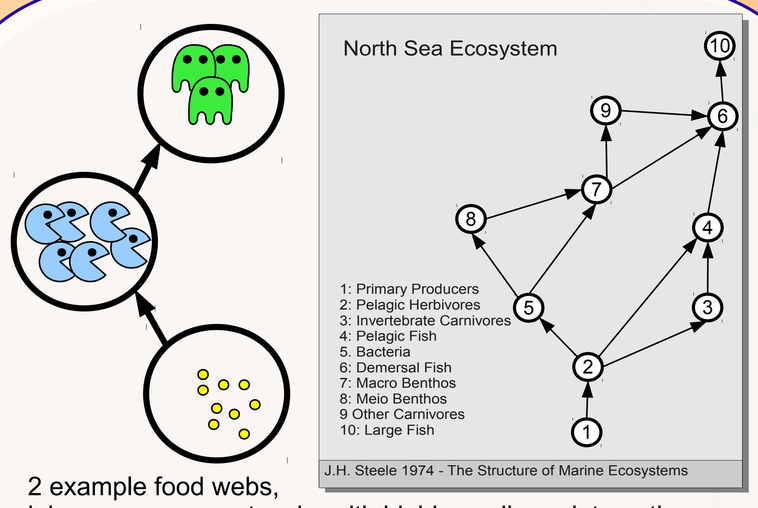
m: Nonlinearity of mortality r: Timescale ratio prey/predator
g: Nonlinearity of predation

From: T. Gross, U. Feudel 2008

Points in parameter space (m,g,r) separated by bifurcation surfaces have qualitatively different dynamics.

Below the blue surface, all combinations admit a stable steady state that is lost in a Hopf bifurcation if g increases across the blue surface.

What is a food web?



2 example food webs, inhomogeneous networks with highly nonlinear interactions

Methods

$$\frac{\mathrm{d}X}{\mathrm{d}t} = \underbrace{G(\vec{X})}_{\text{gain}} - \underbrace{L(\vec{X})}_{\text{loss}}$$

►Model: X is the abundance of the species. Gain and loss result from, mortality, reproduction and predation.

► Assumption: Coexistence of species in a food web requires a non-zero steady state.

►Linearisation: Generalized model Jacobian matrix near coexistence states.

$$\alpha_i = \frac{G_i^*}{X_i^*}$$

►Parameters: Biomass turnover rates (α) and interaction elasticities (μ).

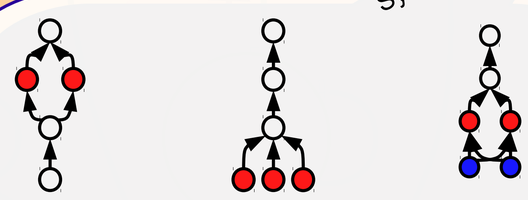
$$\mu_{i,j} = \frac{\partial \log(G_i^*)}{\partial \log(X_j^*)}$$

►Dynamics: Analyse dynamical modes corresponding to the eigenvalues and eigenvectors of the Jacobian matrix.

Modes?

The dynamics is a superposition of dynamical modes, each of the form $\exp(\lambda t)\, v$
λ: eigenvalue of the Jacobian matrix
V: eigenvector.

Symmetries?



3 Examples for Symmetries:
Nodes with the same colour can be exchanged without changing the network; they are symmetric.

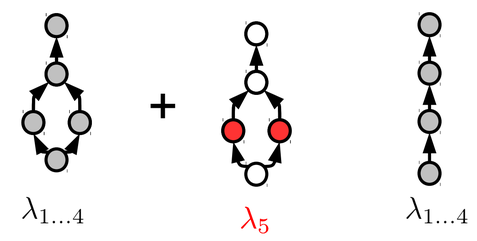
Results

Symmetries generate localized modes.

►Localized modes depend only on the symmetry. They are meso-scale effects, independent from the remaining network.

►Non-localized modes are constant on the symmetry. They are independent from symmetry-internal changes.

►Spectra with and without symmetry differ only by the localized eigenvalues.



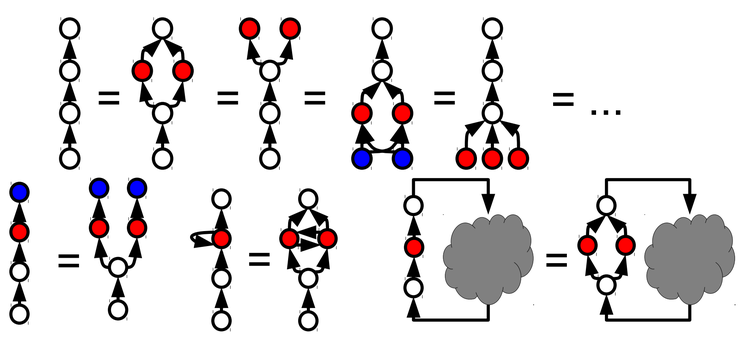
2 Food Web examples and their eigenvalues of the Jacobian matrix.

Implications

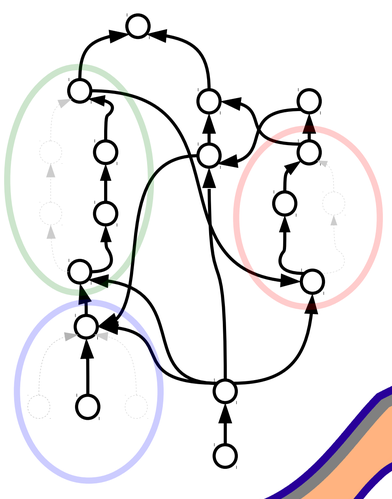
Localized modes describe either a stable or an unstable coexistence.

Stable Modes

►Dynamics has no significant global effect.
This explains the equivalence of the observed dynamics.



►It can be removed.
This removal reduces the number of nodes in the model.



Unstable Modes

►Occurrence of the motif implies dynamical instability of the species coexistence, regardless of the embedding network.

►Characterizable without global methods.

►Generally lead to oscillations and species extinctions in the food web.

►Despite the local origin effects may spread into the entire network. They are a global effect with a meso-scale origin.

Outlook

►Generalizations to other motifs.

►Refine the implications of meso-scale dynamics in systems other than food webs.

►Can failures in real-world systems be traced to the meso-scale?

Read More:

Meso-scale symmetries explain the dynamical equivalence of food webs
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